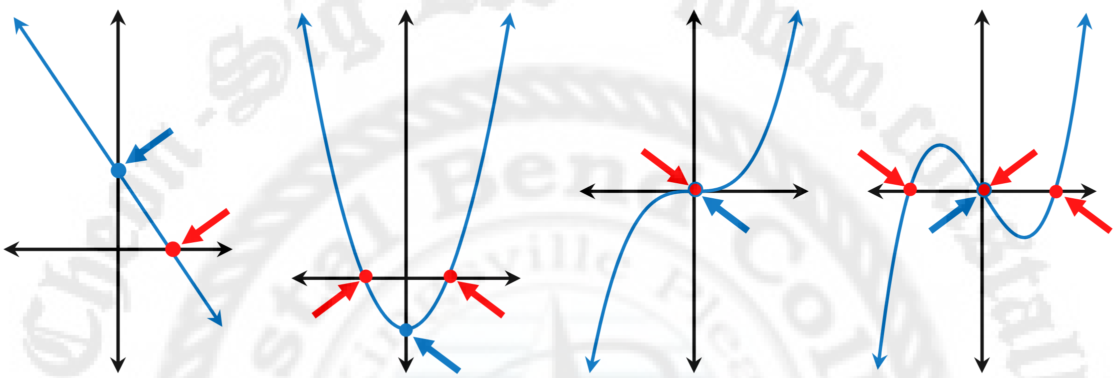
Section 2.1 Intercepts; Symmetry; Graphing Key Equations

Intercepts:

- An **intercept** is the point at which a graph crosses or touches the coordinate axes.
- ***x*–intercept** is
  1. The point where the line crosses (or intercepts) the ***x*-axis**.
  2. The *x*-coordinate of a point at which the graph crosses or touches the *x*-axis.
  3. The x-intercepts of the graph of an equation are those x-values for which ***y* = 0**.
- ***y*–intercept** is
  1. The point where the line crosses (or intercepts) the ***y*-axis**.
  2. The *y*-coordinate of a point at which the graph crosses or touches the *y*-axis.
  3. The *y*-intercepts of the graph of an equation are those *y*-values for which ***x* = 0**.



Notice that a point could be both the *x*-intercept and *y*-intercept.

Finding *x*-intercepts	Finding *y*-intercepts
➢ Step 1: Substitute 0 for *y* or $f(x)$. ➢ Step 2: Solve for the *x* variable.	➢ Step 1: Substitute 0 for *x*. ➢ Step 2: Solve for the *y* variable.

Axis of symmetry is a line of symmetry for a graph

- If a graph has the **x-axis symmetry**, then $f(x) = -f(x)$.
  - Step 1: Replace *y* by – *y* in the equation.
  - Step 2: Solve for *y*.
  - Step 3: The graph of the equation in the step 2 is **symmetric with respect to the *x*-axis** if the equivalent equation results.
- If a graph has the *y*-axis symmetry, then $f(x) = f(-x)$.
  - Step 1: Replace *x* by – *x* in the equation.
  - Step 2: Solve for *y*.
  - Step 3: The graph of the equation in the step 2 is **symmetric with respect to the *y*-axis** if the equivalent equation results
- If a graph has the origin symmetry, then $f(-x) = -f(-x)$
  - Step 1: Replace *x* by – *x* and *y* by – *y* in the equation.
  - Step 2: Solve for *y*.
  - Step 3: The graph of the equation in the step 2 **is symmetric with respect to the origin** if the equivalent equation results

Section 2.1 Intercepts; Symmetry; Graphing Key Equations

Exercises

1. Complete the sentence below.

The points, if any, at which a graph crosses or touches the coordinate axes are called the ________.

The points, if any, at which a graph crosses or touches the coordinate axes are called the [▾]

- x-intercepts.
- intercepts.
- zeros (or roots).
- y-intercepts.

2. Fill in the blank.

The x-intercepts of the graph of an equation are those x-values for which ________.

The x-intercepts of the graph of an equation are those x-values for which $y = 0$.

(Solution 2)
The x-intercepts of the graph of an equation are those x-values for which y = 0.
The y-intercepts of the graph of an equation are those y-values for which x = 0.

3. Complete the sentence below.

If for every point (x,y) on the graph of an equation the point $(-x,y)$ is also on the graph, then the graph is symmetric with respect to the ________.

If for every point (x,y) on the graph of an equation the point $(-x,y)$ is also on the graph, then the graph is symmetric with respect to the y-axis.

(Solution 3) For every point (x, y);
Symmetric with respect to the y-axis is $(-x, y)$
Symmetric with respect to the x-axis is $(x, -y)$
Symmetric with respect to the origin is $(-x, -y)$

4. Fill in the blank.

If the graph of an equation is symmetric with respect to the y-axis and -4 is an x-intercept of this graph, then ________ is also an x- intercept.

If the graph of an equation is symmetric with respect to the y-axis and -4 is an x-intercept of this graph, then 4 is also an x- intercept. (Type an integer or a fraction.)

(Solution 4) -4 is an x-intercept of this graph $\Leftrightarrow (-4, 0)$
Symmetric with respect to the y-axis is $(4, 0)$
Symmetric with respect to the x-axis is $(-4, 0)$
Symmetric with respect to the origin is $(4, 0)$

5. Fill in the blank.

If the graph of an equation is symmetric with respect to the origin and $(3,-4)$ is a point on the graph, then ________ is also a point on the graph.

If the graph of an equation is symmetric with respect to the origin and $(3,-4)$ is a point on the graph, then $(-3,4)$ is also a point on the graph.

(Solution 5) $(-3,4)$
If the graph is symmetric with respect to the y-axis, x-value changes, so $(3,4)$
If the graph is symmetric with respect to the x-axis, y-value changes, so $(-3,-4)$
If the graph is symmetric with respect to the origin, both x-value and y-value change, so $(3,-4)$

6. Decide whether the following statement is true or false.

To find y-intercepts of the graph of an equation, let $x=0$ and solve for y.

Choose the correct answer below.

- False
- ✔ True

(Solution 6)
To find x-intercepts of the graph of an equation, let y = 0 and solve for x.
To find y-intercepts of the graph of an equation, let x = 0 and solve for y.

7. State whether the following statement is true or false.

The y-coordinate of a point at which the graph crosses or touches the x-axis is an x-intercept.

Choose the correct answer below.

- True
- ✔ False

(Solution 7)
The x-coordinate of a point at which the graph crosses or touches the x-axis is an x-intercept.
The y-coordinate of a point at which the graph crosses or touches the y-axis is a y-intercept.

8. Decide whether the following statement is true or false.

If a graph is symmetric with respect to the x-axis, then it cannot be symmetric with respect to the y-axis.

Choose the correct answer below.

- ✔ False
- True

(Solution 8)
The statement is false because a graph of a circle is symmetric with respect to the x-axis, y-axis, and origin.

9. Find the intercepts and graph the equation by plotting points.

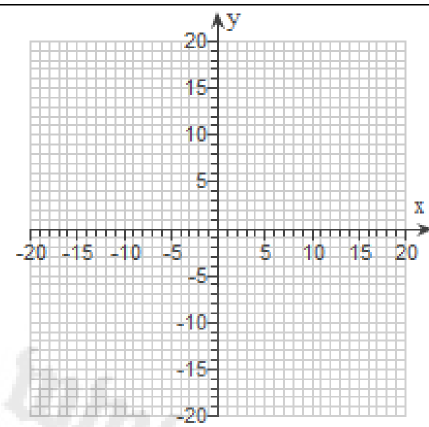
$y = 2x^2 - 8$

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

○A. The intercept(s) is/are ☐.
(Type an ordered pair.)
(Use a comma to separate answers as needed.)

○B. There are no intercepts.

Use the graphing tool to graph the equation.



(Solution 9)

***x*-intercept**

Step 1: Substitute 0 for y.

$y = 2x^2 - 8$

$0 = 2x^2 - 8$

Step 2: Solve for x.

$2x^2 - 8 = 0$

$+8 \quad +8$

$2x^2 = 8$

$\frac{2x^2}{2} = \frac{8}{2}$

$x^2 = 4$

$\sqrt{x^2} = \pm\sqrt{4}$

$x = \pm 2$

So, x-ints are $(2,0), (-2,0)$

***y*-intercept**

Step 1: Substitute 0 for x.

$y = 2x^2 - 8$

$y = 2(0)^2 - 8$

Step 2: Solve for y.

$y = 2(0)^2 - 8$

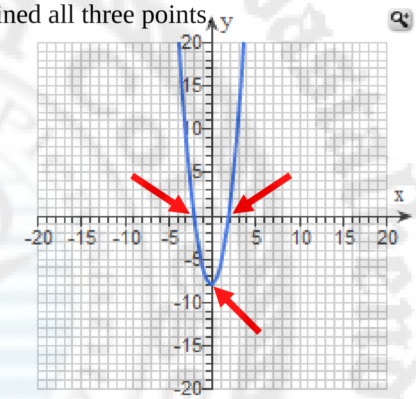
$y = 2(0) - 8$

$y = 0 - 8$

$y = -8$

So, y-intercept is $(0, -8)$

Therefore, the intercepts are $(2,0), (-2,0), (0,-8)$ and the graph is shown as below when you plot the obtained all three points.



10. Plot the point. Then plot the point that is symmetric to it with respect to **(a)** the x-axis; **(b)** the y-axis; **(c)** the origin.

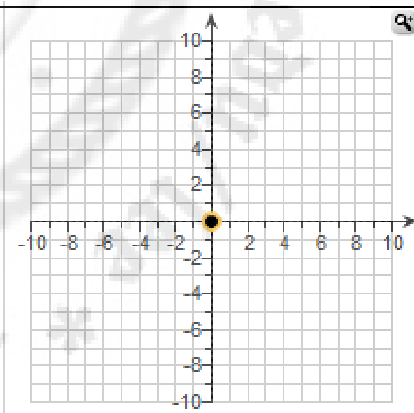
(3,2)

Plot the point (3,2).

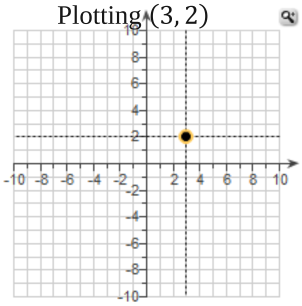
(a) Plot the point that is symmetric to (3,2) with respect to the x-axis.

(b) Plot the point that is symmetric to (3,2) with respect to the y-axis.

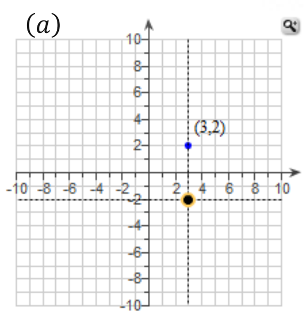
(c) Plot the point that is symmetric to (3,2) with respect to the origin.



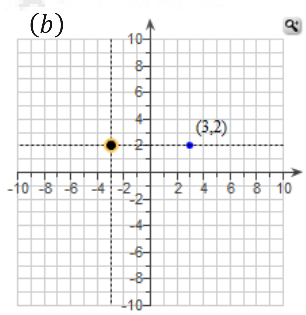
Plotting $(3,2)$



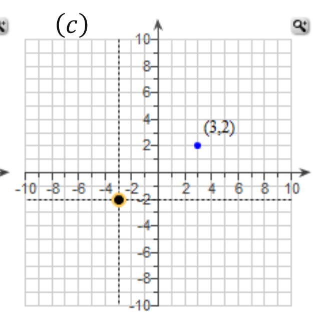
(a)



(b)



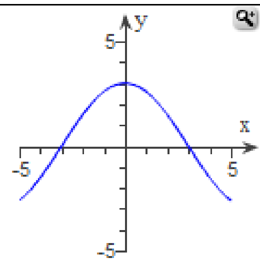
(c)



11. The graph of an equation is shown on the right.

(a) Find the intercepts.

(b) Indicate whether the graph is symmetric with respect to the x-axis, the y-axis, or the origin.



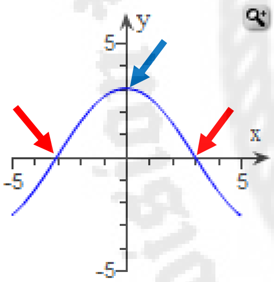
(a) Select the correct choice below and fill in any answer boxes within your choice.

- ✔ A. The intercept(s) is/are $(-3,0),(3,0),(0,3)$.
  (Type an ordered pair. Use a comma to separate answers as needed.)
- B. There are no intercepts.

(b) Is the graph symmetric with respect to the x-axis, the y-axis, or the origin?

- A. The graph is symmetric with respect to the origin.
- B. The graph is symmetric with respect to the x-axis.
- ✔ C. The graph is symmetric with respect to the y-axis.
- D. The graph has no symmetry.

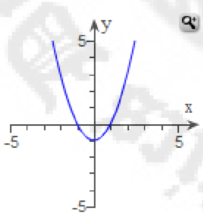
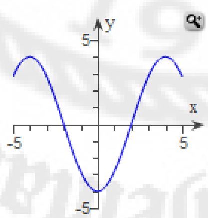
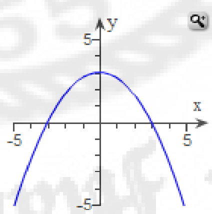
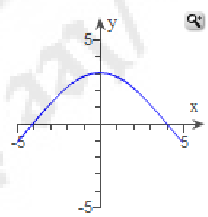
(Solution 11)



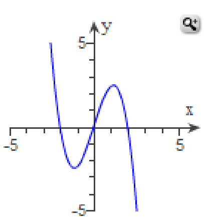
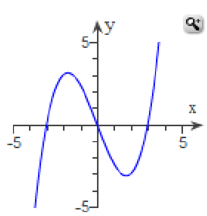
(*a*) x-intercepts are $(-3,0)$ and $(3,0)$
y-intercept is $(0,3)$
Thus, intercepts are $(-3,0), (3,0), (0,3)$

(*b*) Actually, we do not have enough information to decide the symmetry. We must assume that the graph has the symmetry.

y-axis symmetry:

Origin symmetry:

12. List the intercepts and test for symmetry.

$y = x^4 - 10x^2 - 96$

List the x-intercept(s), if there are any. Select the correct choice below and fill in any answer boxes within your choice.

- ✔ A. The x-intercept(s) is/are $(-4,0),(4,0)$.
  (Type an ordered pair.)
  (Use a comma to separate answers as needed.)
- B. There is no x-intercept.

List the y-intercept(s), if there are any. Select the correct choice below and fill in any answer boxes within your choice.

- ✔ A. The y-intercept(s) is/are $(0,-96)$.
  (Type an ordered pair.)
  (Use a comma to separate answers as needed.)
- B. There is no y-intercept.

Is the graph of the equation symmetric with respect to the x-axis?

Yes ✔ No

Is the graph of the equation symmetric with respect to the y-axis?

No ✔ Yes

Is the graph of the equation symmetric with respect to the origin?

Yes ✔ No

(Solution 12)

Finding *x*-intercept(s)

Substitute 0 for *y*, then solve for *x*.

$y = x^4 - 10x^2 - 96$

$0 = x^4 - 10x^2 - 96$

Let m be x^2, then $m \geq 0$ and we have

$(x^2)^2 - 10x^2 - 96 = 0$

$m^2 - 10m - 96 = 0$

By *ac*-method, we have

$(m+6)(m-16) = 0$

$m + 6 = 0$ or $m - 16 = 0$

$\underline{\quad -6 \quad -6}$ $\qquad$ $\underline{\quad +16 \quad +16}$

$m = -6$ $\qquad$ $m = 16$

Since $m = x^2$, *m* is a positive for all *x*. Thus, discard the proposed solution – 6.

$m = 16 \Rightarrow \quad x^2 = 16$

$\sqrt{x^2} = \pm\sqrt{16}$

$x = \pm 4$

Therefore, the *x*-intercepts are $(4,0), (-4,0)$

Finding *y*-intercept(s)

Substitute 0 for *x*, then solve for *y*.

$y = x^4 - 10x^2 - 96$

$y = (0)^4 - 10(0)^2 - 96$

$y = -96$

Therefore, the *y*-intercept is $(0,-96)$

***x*-axis symmetry**

Replace *y* by –*y*, then solve for *y*.

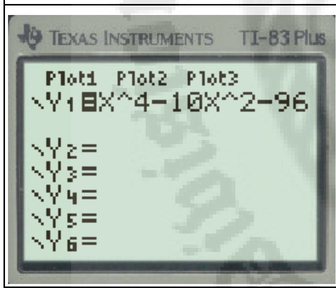
$y = x^4 - 10x^2 - 96$

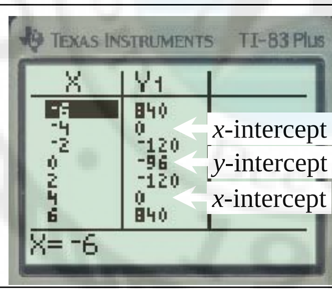
$-y = x^4 - 10x^2 - 96$

$\frac{-y}{-1} = \frac{x^4}{-1} + \frac{-10x^2}{-1} + \frac{-96}{-1}$

$y = -x^4 + 10x^2 + 96$

$y = -x^4 + 10x^2 + 96$ is different from the original equation, so the graph is not symmetric with respect to the *x*-axis.





***y*-axis symmetry**

Replace *x* by –*x*, then solve for *y*.

$y = x^4 - 10x^2 - 96$

$y = (-x)^4 - 10(-x)^2 - 96$

$y = x^4 - 10x^2 - 96$

The equivalent equation results, so the graph has the *y*-axis symmetry.

Origin symmetry

Replace *x* by –*x* and *y* by –*y*, then solve for *y*.

$y = x^4 - 10x^2 - 96$

$-y = (-x)^4 - 10(-x)^2 - 96$

$-y = x^4 - 10x^2 - 96$

$\frac{-y}{-1} = \frac{x^4}{-1} + \frac{-10x^2}{-1} + \frac{-96}{-1}$

$y = -x^4 + 10x^2 + 96$

$y = -x^4 + 10x^2 + 96$ is different from the original equation, so the graph is not symmetric with respect to the origin.

13.
a) Find the intercepts of the equation.
b) Test the equation for symmetry with respect to the x-axis, the y-axis, and the origin.
c) Graph the equation by plotting points.

$y = x^3 - 7x$

List the x-intercept(s). Select the correct choice below and fill in any answer boxes within your choice.

✓ A. The x-intercept(s) is/are $(0,0),(\sqrt{7},0),(-\sqrt{7},0)$.
(Type an ordered pair. Use a comma to separate answers as needed.)
(Type exact answers for each coordinate, using radicals as needed.)

B. There is no x-intercept.

List the y-intercept(s). Select the correct choice below and fill in any answer boxes within your choice.

✓ A. The y-intercept(s) is/are $(0,0)$.
(Type an ordered pair. Use a comma to separate answers as needed.)
(Type exact answers for each coordinate, using radicals as needed.)

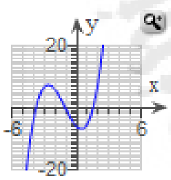
B. There is no y-intercept.

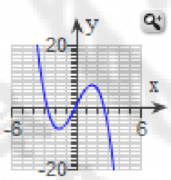
Is the graph of the equation symmetric with respect to the x-axis?
✓ No Yes

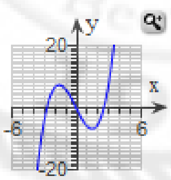
Is the graph of the equation symmetric with respect to the y-axis?
Yes ✓ No

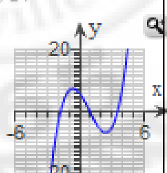
Is the graph of the equation symmetric with respect to the origin?
No ✓ Yes

Graph the function. Choose the correct graph below.

A.


B.


✓ C.


D.


(Solution 13)

x-intercept(s)

Substitute 0 for *y*, then solve for *x*.

$$y = x^3 - 7x$$
$$0 = x^3 - 7x$$
$$0 = x(x^2 - 7)$$
$$x = 0 \quad \text{or} \quad x^2 - 7 = 0$$
$$\quad +7 \quad +7$$
$$x^2 = 7$$
$$\sqrt{x^2} = \pm\sqrt{7}$$
$$x = \pm\sqrt{7}$$

Therefore, the *x*-intercepts are

$(0,0)$, $(\sqrt{7},0)$ and $(-\sqrt{7},0)$

y-intercept(s)

Substitute 0 for *x*, then solve for *y*.

$$y = x^3 - 7x$$
$$y = 0^3 - 7(0)$$
$$y = 0$$

Thus, the *y*-intercept is $(0,0)$

Therefore, the intercepts are

$(0,0)$, $(\sqrt{7},0)$, and $(-\sqrt{7},0)$

***x*-axis symmetry**

Replace *y* by –*y*, then solve for *y*.

$$y = x^3 - 7x$$
$$-y = x^3 - 7x$$
$$\frac{-y}{-1} = \frac{x^3}{-1} + \frac{-7x}{-1}$$
$$y = -x^3 + 7x$$

$y = -x^3 + 7x$ is different from the original equation, so the graph is **NOT** symmetric with respect to the *x*-axis.

***y*-axis symmetry**

Replace *x* by –*x*, then solve for *y*.

$$y = x^3 - 7x$$
$$y = (-x)^3 - 7(-x)$$
$$y = -x^3 + 7x$$

$y = -x^3 + 7x$ is different from the original equation, so the graph is **NOT** symmetric with respect to the *y*-axis.

Origin symmetry

Replace *x* by –*x* and *y* by –*y*, then solve for *y*.

$$y = x^3 - 7x$$
$$-y = (-x)^3 - 7(-x)$$
$$-y = -x^3 + 7x$$
$$\frac{-y}{-1} = \frac{-x^3}{-1} + \frac{7x}{-1}$$
$$y = x^3 - 7x$$

The equivalent equation results, so the graph has the origin symmetry.

Graph the function.

Step 1: Plot all intercepts obtained.

Step 2: Find behavior of the end points.

Section 2.1 Intercepts; Symmetry; Graphing Key Equations

Solving Quadratic Equations

- **By Factoring**
- **By Square Root Property and Completing the square**
- **By Quadratic Formula**

Solving Quadratic Equations by Factoring

- Step 1: Factor out GCF, if possible
- Step 2: Determine the number of terms in the polynomial
- Step 3: Determine a factoring technique as follows:

  If the given polynomial is a **Binomial**, factoring by one of the following

  1. $a^2 - b^2 = (a+b)(a-b)$
  2. $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$
  3. $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

  If the given polynomial is a **Trinomial**, factoring by ***ac*-method**

  If the given polynomial has 4 or more terms, factoring by **Grouping**

Greatest Common Factor (GCF):

- The GCF is an expression of the highest degree that divides each term of the polynomial.
- The variable part of the greatest common factor always contains **the smallest power** of a variable that appears in all terms of the polynomial.

Finding GCF

- Step 1: Find the prime factorization of all integers and integer coefficients
- Step 2: List all the factors that are common to all terms, including variables
- Step 3: Choose the smallest power for each factor that is common to all terms
- Step 4: Multiply these powers to find the GCF

  NOTE: If there is no common prime factor or variable, then the GCF is 1

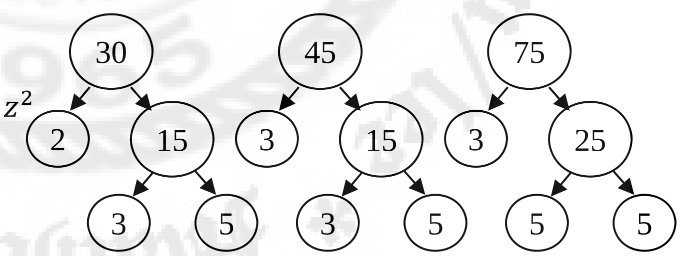
(Example) Find the GCF for the following set of algebraic terms: $\{30x^4y, 45x^3y, 75x^5y^2\}$

$30x^4y^3z = 2 \cdot 3 \cdot 5 \cdot x^4 \cdot y^3 \cdot z$

$45x^3y = 3 \cdot 3 \cdot 5 \cdot x^3 \cdot y = 3^2 \cdot 5 \cdot x^3 \cdot y$

$75x^5z^2 = 3 \cdot 5 \cdot 5 \cdot x^5 \cdot z^2 = 3 \cdot 5^2 \cdot x^3 \cdot z^2$

Therefore, GCF is $3 \cdot 5 \cdot x^3 = 15x^3$



Section 2.1 Intercepts; Symmetry; Graphing Key Equations

Factoring by '*ac*-method' is used for trinomials and some binomials

- Factoring *ac*-method will be discussed with examples

Examples for **positive *ac***

1. Factor the given polynomial $x^2 + 7x + 10$

Step 1: Factor out GCF. GCF = 1, so DONE!

Step 2: Find *a*, *b*, *c*

$a = 1,\ b = 7,\ c = 10$

Step 3: Find *ac*

$ac = 1 \cdot 10 = 10$

Step 4: Find positive factors of *ac* whose products is *ac* and whose sum is '*b*'

Positive factors of 10: $(1 \times 10), (2 \times 5)$

Since *ac* is positive, there are two case;

Case 1: Both factors are positive

Case 2: Both factors are negative

Since '*b*' is positive, both factors are positive

Thus, possible factors are

$(1 \times 10), (\mathbf{2} \times \mathbf{5})$

Step 5: Rewrite the middle term, ***bx***, using the two numbers found in step 4 as coefficients. Then factoring by grouping.

$$\begin{aligned} x^2 + 7x + 10 &= x^2 + \mathbf{2x} + \mathbf{5x} + 10 \\ &= (x^2 + 2x) + (5x + 10) \\ &= x(\mathbf{x+2}) + 5(\mathbf{x+2}) \\ &= (\mathbf{x+2})(x+5) \end{aligned}$$

2. Factor the given polynomial $x^2 - 5x + 4$

Step 1: Factor out GCF. GCF = 1, so DONE!

Step 2: Find *a*, *b*, *c*

$a = 1,\ b = -5,\ c = 4$

Step 3: Find *ac*

$ac = 1 \cdot 4 = 4$

Step 4: Find positive factors of *ac* whose products is *ac* and whose sum is '*b*'

Positive factors of 4: $(1 \times 4), (2 \times 2)$

Since *ac* is positive, there are two case;

Case 1: Both factors are positive

Case 2: Both factors are negative

Since '*b*' is negative, both factors are negative

Thus, possible factors are

$(-\mathbf{1}) \times (-\mathbf{4}), (-2) \times (-2)$

Step 5: Rewrite the middle term, ***bx***, using the two numbers found in step 4 as coefficients. Then factoring by grouping.

$$\begin{aligned} x^2 - 5x + 4 &= x^2 - \mathbf{x} - \mathbf{4x} + 4 \\ &= (x^2 - x) - (4x - 4) \\ &= x(\mathbf{x-1}) - 4(\mathbf{x-1}) \\ &= (\mathbf{x-1})(x-4) \end{aligned}$$

3. Factor the given polynomial $4x^2 + 22x + 10$

Step 1: Factor out GCF. GCF = 2

$4x^2 + 22x + 10 = 2(2x^2 + 11x + 5)$

Step 2: Find *a*, *b*, *c*

$a = 2,\ b = 11,\ c = 5$

Step 3: Find *ac*

$ac = 2 \cdot 5 = 10$

Step 4: Find positive factors of *ac* whose products is *ac* and whose sum is '*b*'

Positive factors of 10: $(1 \times 10), (2 \times 5)$

Since *ac* is positive, there are two case;

Case 1: Both factors are positive

Case 2: Both factors are negative

Since '*b*' is positive, both factors are positive

Thus, possible factors are

$(\mathbf{1} \times \mathbf{10}), (2 \times 5)$

Step 5: Rewrite the middle term, ***bx***, using the two numbers found in step 4 as coefficients. Then, factoring by grouping.

$$\begin{aligned} 2(2x^2 + 11x + 5) &= 2(2x^2 + \mathbf{x} + \mathbf{10x} + 5) \\ &= 2[(2x^2 + x) + (10x + 5)] \\ &= 2[x(\mathbf{2x+1}) + 5(\mathbf{2x+1})] \\ &= 2(\mathbf{2x+1})(x+5) \end{aligned}$$

4. Factor the given polynomial $8x^2 - 36x + 28$

Step 1: Factor out GCF. GCF = 4

$8x^2 - 36x + 28 = 4(2x^2 - 9x + 7)$

Step 2: Find *a*, *b*, *c*

$a = 2,\ b = -9,\ c = 7$

Step 3: Find *ac*

$ac = 2 \cdot 7 = 14$

Step 4: Find positive factors of *ac* whose products is *ac* and whose sum is '*b*'

Positive factors of 14: $(1 \times 14), (2 \times 7)$

Since *ac* is positive, there are two case;

Case 1: Both factors are positive

Case 2: Both factors are negative

Since '*b*' is negative, both factors are negative

Thus, possible factors are

$(-1) \times (-14), (-\mathbf{2}) \times (-\mathbf{7})$

Step 5: Rewrite the middle term, ***bx***, using the two numbers found in step 4 as coefficients. Then, factoring by grouping.

$$\begin{aligned} 4(2x^2 - 9x + 7) &= 4(2x^2 - \mathbf{2x} - \mathbf{7x} + 7) \\ &= 4[(2x^2 - 2x) - (7x - 7)] \\ &= 4[2x(x-1) - 7(x-1)] \\ &= 4(x-1)(2x-7) \end{aligned}$$

Section 2.1 Intercepts; Symmetry; Graphing Key Equations

Examples for **negative *ac***

1. Factor the given polynomial $x^2 + 3x - 10$

   Step 1: Factor out GCF. GCF = 1, so DONE!

   Step 2: Find *a*, *b*, *c*

   $a = 1,\ b = 3,\ c = -10$

   Step 3: Find *ac*

   $ac = 1 \cdot (-10) = -10$

   Step 4: Find positive factors of *ac* whose products is *ac* and whose sum is '*b*'

   Positive factors of 10: $(1 \times 10), (2 \times 5)$

   Since *ac* is negative, one factor is positive and the other is negative

   Since '*b*' is positive, bigger factor is positive

   Thus, possible factors are

   $(-1) \times 10, (\mathbf{-2}) \times \mathbf{5}$

   Step 5: Rewrite the middle term, ***bx***, using the two numbers found in step 4 as coefficients. Then factoring by grouping.

   $$\begin{aligned} x^2 + 3x - 10 &= x^2 \mathbf{- 2x} \mathbf{+ 5x} - 10 \\ &= (x^2 - 2x) + (5x - 10) \\ &= x(\mathbf{x - 2}) + 5(\mathbf{x - 2}) \\ &= (\mathbf{x - 2})(x + 5) \end{aligned}$$

2. Factor the given polynomial $x^2 - 3x - 10$

   Step 1: Factor out GCF. GCF = 1, so DONE!

   Step 2: Find *a*, *b*, *c*

   $a = 1,\ b = -3,\ c = -10$

   Step 3: Find *ac*

   $ac = 1 \cdot (-10) = -10$

   Step 4: Find positive factors of *ac* whose products is *ac* and whose sum is '*b*'

   Positive factors of 10: $(1 \times 10), (2 \times 5)$

   Since *ac* is negative, one factor is positive and the other is negative

   Since '*b*' is negative, bigger factors is negative

   Thus, possible factors are

   $1 \times (-10), \mathbf{2} \times (\mathbf{-5})$

   Step 5: Rewrite the middle term, ***bx***, using the two numbers found in step 4 as coefficients. Then factoring by grouping.

   $$\begin{aligned} x^2 + 3x - 10 &= x^2 \mathbf{+ 2x} \mathbf{- 5x} - 10 \\ &= (x^2 + 2x) - (5x + 10) \\ &= x(\mathbf{x + 2}) - 5(\mathbf{x + 2}) \\ &= (\mathbf{x + 2})(x - 5) \end{aligned}$$

3. Factor the given polynomial $4x^2 + 22x - 10$

   Step 1: Factor out GCF. GCF = 2

   $4x^2 + 22x + 10 = 2(2x^2 + 9x - 5)$

   Step 2: Find *a*, *b*, *c*

   $a = 2,\ b = 11,\ c = -5$

   Step 3: Find *ac*

   $ac = 2 \cdot (-5) = -10$

   Step 4: Find positive factors of *ac* whose products is *ac* and whose sum is '*b*'

   Positive factors of 10: $(1 \times 10), (2 \times 5)$

   Since *ac* is negative, one factor is positive and the other is negative

   Since '*b*' is positive, bigger factor is positive

   Thus, possible factors are

   $(\mathbf{-1}) \times \mathbf{10}, (-2) \times 5$

   Step 5: Rewrite the middle term, ***bx***, using the two numbers found in step 4 as coefficients. Then, factoring by grouping.

   $$\begin{aligned} 2(2x^2 + 11x + 5) &= 2(2x^2 \mathbf{- x} \mathbf{+ 10x} + 5) \\ &= 2[(2x^2 - x) + (10x + 5)] \\ &= 2[x(\mathbf{2x - 1}) + 5(\mathbf{2x - 1})] \\ &= 2(\mathbf{2x - 1})(x + 5) \end{aligned}$$

4. Factor the given polynomial $6x^2 - 15x - 21$

   Step 1: Factor out GCF. GCF = 3

   $6x^2 - 27x + 21 = 3(2x^2 - 5x - 7)$

   Step 2: Find *a*, *b*, *c*

   $a = 2,\ b = -5,\ c = -7$

   Step 3: Find *ac*

   $ac = 2 \cdot (-7) = 14$

   Step 4: Find positive factors of *ac* whose products is *ac* and whose sum is '*b*'

   Positive factors of 14: $(1 \times 14), (2 \times 7)$

   Since *ac* is negative, one factor is positive and the other is negative

   Since '*b*' is negative, bigger factors is negative

   Thus, possible factors are

   $1 \times (-14), \mathbf{2} \times (\mathbf{-7})$

   Step 5: Rewrite the middle term, ***bx***, using the two numbers found in step 4 as coefficients. Then, factoring by grouping.

   $$\begin{aligned} 3(2x^2 - 5x - 7) &= 3(2x^2 \mathbf{+ 2x} \mathbf{- 7x} - 7) \\ &= 3[(2x^2 + 2x) - (7x + 7)] \\ &= 3[2x(x + 1) - 7(x + 1)] \\ &= 3(x + 1)(2x - 7) \end{aligned}$$

Square Root Property

If $x^2 = a$, then $x = \pm\sqrt{a}$ where *a* is a nonzero real number.

Note: $x = \pm\sqrt{a}$ indicates that $x = \sqrt{a}$ or $x = -\sqrt{a}$